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ABSTRACT

Thirteen research reports related to mathematics education are abstracted and analyzed. Two of the reports deal with acquisition of number concepts; three deal with instructional techniques; and one each pertains to diagnosis, logical connectives, teacher behaviors, spatial preferences, mathematical language, formal thought, and curriculum evaluation. Another reviewed document is concerned with spatial and geometric concepts; it contains eleven research papers and three review articles. Research related to mathematics education reported in RIE and CIJE between April and June 1979 is listed. (MK)

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Fall 1979

Bessot, Annie and Comiti, Claude. UNE ETUDE SUR L'APPROCHE DU NOMBRE PAR L'ELEVE DU COURS PREPARATOIRE. Educational Studies in Mathematics 9: 17-39; February 1978.

Abstract and comments prepared for I.M.E. by GERALD A. GOLDIN, Northern Illinois University.

1. Purpose

This study explores some of the processes used by children in understanding natural numbers. The children are of ages 6 and 7, enrolled in the *Cours Preparatoire* (C.P.). The following questions are asked: (1) What procedures are spontaneously used in order to construct a set equivalent to a given set; or, alternatively, to construct a set having more elements than a given set? (2) Are some procedures more frequently and/or more successfully employed than others? (3) How are the answers to these questions affected by schooling and by the child's psychogenetic development?

2. Rationale

In discussing the understanding of natural numbers, it is widely recognized that three numerical "domains" may be distinguished, each with its own specific preferred method: (a) whole numbers from 0 to 6, global perception; (b) whole numbers from 7 to 20 or 25, one-to-one correspondence; (c) whole numbers above 20 or 25, correspondence by regrouping (*paquet par paquet*). This study is intended to provide more detail concerning the processes employed in understanding the second of these numerical domains.

3. Research Design and Procedures

Two tasks are used in this study, in individual structured interviews with the children. In Task A, 15 red chips (*jetons*) are spread out on the table by the experimenter. The child has a container with 25 blue chips, and is asked to "Take out as many blue chips as there are red chips." If the child takes all the blue chips, or does not respond to the question (*fait n'importe quoi*), or does nothing at all, the instruction is repeated with the additional phrase, "If you wish, you may touch the red chips." Whatever the outcome, the child is next asked, "Are you sure that you have as many blue chips as red chips?" Those children who are not sure, or who are sure but are in fact wrong, are then asked to check their results (some children check their results spontaneously).

In Task B, 18 yellow chips are placed on the table; the child again has 25 blue chips. This time the instruction is, "Take more blue chips than there are yellow chips." If the child does not respond to the question or does nothing at all, the instruction is repeated with the phrase, "If you wish, you may touch the yellow chips." Whatever the outcome, the child is next asked, "Are you sure that there are more blue chips than yellow chips?" and in the event of a simple "Yes," the child is asked, "How do you know?"

The tasks are accompanied by a record sheet which is filled out during the interview. On this sheet the behavior of the child at each step is classified in one of several possible categories, based on the method employed or the response given to a question. For example, in Task A when the child is asked to "Take as many blue chips as there are red chips," the investigator makes a note as to whether or not the arrangement of red chips is modified. Then the method used by the child is listed as one of the following: counting; correspondence by regrouping (*par paquets*); pairwise correspondence; superposition; arrangement in facing rows; taking all of the blue chips; doing something else (*n'importe quoi*); doing nothing; or other (described). The success or lack of success of the child at each step is also recorded.

The tasks are administered to 121 children in six different C.P. -- four urban, one rural, and one in a small industrial town. Most of the tables report the outcomes descriptively, in terms of the number of children who succeed or fail at each step on each task, the number who employ each strategy on each task, and so forth (see below). Some of the outcomes are examined by means of 2x2 contingency tables, and chi-square tests of significance are used--for example, to determine whether the choice of method (counting *vs.* one-to-one correspondence) influences success on the task, whether initial success is associated with the child's expressed certainty of success, and so forth.

4. Findings

On Task A, 52 of the 121 children succeeded and 69 did not succeed in constructing a set equivalent to the given set. Among the 52 who succeeded, there were 48 who were sure of their results; among the 69 who did not succeed, only 44 were sure of their results. Forty-two of the 69 children were able to correct their initial constructions when asked to verify their results; the remaining 27 may be said to have completely failed the task, either because of complete lack of comprehension of the instruction "as many as" (*"autant"*), or because of the inability to find or to carry out an applicable strategy.

On Task B, 81 children succeeded unambiguously in constructing a set larger than the given set; another 24 satisfied the condition of the task without it being possible to establish whether this was due to global perception or to chance; and 16 failed to construct a set having more elements than the given set. Only 3 of the 52 children who succeeded on Task A failed on Task B; while 18 of the 27 children who failed completely on Task A succeeded on Task B.

On Task A, only 31 of the 121 children spontaneously manipulated the red chips; 21 of these succeeded in the task on the first instruction. Of the 90 children who did not spontaneously manipulate the red chips, only 22 succeeded in the task at this point. The instruction, "If you wish, you may touch the red chips," was only given to the 48 children who evidenced no strategy; 12 of these now manipulated the red chips, and 5 of the 12 succeeded in the task, while of the 36 who still did not manipulate the red chips, only 4 succeeded in the task.* A chi-square test on the 121 subjects indicated that manipulation of the red chips was associated with success on the task at a .01 level of significance.

Similarly on Task B, manipulation of the yellow chips appeared to be associated with success.

On Task A, the following main relevant strategies were employed by a total of 92 children: counting (51), one-to-one correspondence (37), and correspondence by regrouping (4). One-to-one correspondence

*Those who refer to the original paper may wish to note that in Table II (cont.), the headings *oui* and *non* of the columns appear to be reversed.

was further broken down into arrangement in facing rows (8), superposition (5), and other pairwise correspondence (24). Of the 80 students who carried out a verification of their result on Task A, the strategies used were: counting (41), one-to-one correspondence (25), correspondence by regrouping (3), and others (11).

On Task B, 82 children used strategies which could be classified under the above headings, with a slightly different order of frequency: one-to-one correspondence (44), counting (36), correspondence by regrouping (2). In addition, 26 students used the strategy of emptying the whole container of blue chips.

On Task A, there was a relationship between strategy used and success ($p < .10$). Twenty-five of the 37 children who used one-to-one correspondence succeeded on the task, and 25 of the 51 children who used counting succeeded. There was no significant relationship between strategy used and the expression of certainty by the child. There was a significant association ($p < .01$) between success on Task A and the expression of certainty by the child.

Finally, there was found to be a considerable degree of consistency in the strategies used within each task, as well as from Task A to Task B. Fifty-one of the 121 children used the same strategy from beginning to end.

5. Interpretations

The resistance of many children to touching the chips on the table spontaneously is noteworthy. This is a possible consequence of teaching practices, and is especially unfortunate because of the strong association observed between manipulation and success on the tasks.

The children's greater success on Task B may result either (a) from the possibility that the concept "more" is better understood than the concept "as many as," or (b) from learning which takes place during Task A, or (c) from the fact that a processing error (e.g., miscounting) does not lead to failure on Task B as surely as it would on Task A.

A measure of the "operationality" of a method is defined to be the number of children using the method successfully, divided by the total number using the method. One-to-one correspondence has a greater operationality (0.67) than does counting (0.49) for this population, in the given numerical domain (for which one-to-one correspondence was identified in the rationale as the preferred method). Operationalities of methods were not compared for Task B, since here a method could lead to a correct answer even when used incorrectly (e.g., counting).

More children used one-to-one correspondence than used counting on Task B, in contrast to Task A. This may result either (a) from the fact that school instruction stresses counting mainly in the context of determining the cardinality of a given set, or (b) from non-mastery by C.P. children of the order relationship among numbers of this magnitude.

Abstractor's Comments

Among the strengths of this study are (1) its delineation of a set of alternative methods or strategies for carrying out the two tasks, and (2) the creation of a structured interview framework for observing, recording in detail, and comparing the effects of strategy usage. The strategies identified appear to be widely used by children at this age level; thus we have the prospect of comparing their usage frequencies and operationalities for other populations and other numerical domains.

An important question in evaluating this study is that of the reliability of the classification of behaviors by the experimenter. We are not provided with detailed definitions of the behaviors which were recorded—for example, without a definition for "manipulation" of the chips on the table, it is unclear whether children who counted those chips by pointing at them were considered to have "manipulated" them. There is also no mention of any attempt to cross-check the classification of behaviors; for example, by having them scored independently by more than one observer during the course of the interview. From the information provided it would probably be quite difficult to replicate this study with confidence that the same strategies were being recorded.

The major difficulty with the study is one's inability to draw conclusions from the data. In this experiment we have a single pair of tasks in a fixed sequence, and a mixed population of children with a variety of instructional histories. Consequently, when one strategy is observed to be used more frequently or more effectively than another, it is *a priori* impossible to determine whether the effect is due to (a) characteristics of the tasks or their sequence, or (b) characteristics of the population, such as age or developmental level, or (c) the instructional histories of the children.

To elaborate on this point, consider the finding that counting was less operational than one-to-one correspondence. On the surface this appears to be a statement about the children's development. However, in a strictly logical sense, one can argue that counting is the performance of two successive one-to-one correspondences: first the correspondence of the chips on the table with the standard sounds for the numbers, then the correspondence of the sounds with the chips taken from the child's container. The probability of success in performing the operation twice might be expected approximately to equal the square of the probability of success in performing the operation once; and indeed 0.49 is quite close to the square of 0.67. This kind of analysis seeks to attribute the finding to the intrinsic structure of the task. A compelling argument could also be made for attributing the finding to the nature of the school instruction which the children had received up to the point in the year when the study was conducted (the end of the second trimester of the school year).

The authors of the paper are careful not to draw unjustified conclusions. More meaningful interpretations of the findings would have been possible if the experimental design had permitted some variables to take on a range of values while others were held constant. For instance, the tasks could have been administered in the opposite order (Task B first) to half of the population; or an equal number of children could have performed the same tasks but with a larger number of chips; or the identical procedures could have been followed with children at distinct levels of schooling. The absence of any such experimental variable limits the conclusions which can be drawn. Despite this severe limitation, the report is instructive and informative, and should motivate additional research into the factors influencing children's strategies.

Brown, John Seely and Burton, Richard R. DIAGNOSTIC MODELS FOR PROCEDURAL BUGS IN BASIC MATHEMATICS SKILLS. Cambridge, Massachusetts: Bolt, Beranek and Newman, Inc., December 1977. ERIC: ED 159 036.

Abstract and comments prepared for I.M.E. by GEORGE W. BRIGHT, Northern Illinois university.

1. Purpose

The report presents an application of a computer-based procedure for developing diagnostic models of children's errors with computational algorithms. Subtraction of whole numbers is the application chosen. The report is not primarily an empirical investigation, but rather is a development of a tool for diagnosing students' difficulties.

2. Rationale

The term, diagnostic model, is used "to mean a representation of a student's procedural knowledge or skill that depicts his internalization of a skill as a variant of a correct version of that skill." A diagnostic model for a student's errors is identified by computer analysis of the errors. The computation algorithm (e.g., subtraction) is analyzed into subskills (subprograms) which can be called up as needed. This list of subprograms is then expanded to include erroneous variants (bugs) which may be used by students to obtain incorrect answers. These bugs may be identified through theoretical analysis or analysis of student's work. A student's problem can be diagnosed by comparing her/his answers to those produced by the bugs so identified. The bug that reproduces the student's answers would model a probable cause of the student's difficulty.

Several levels of variants can be handled in this model. Simple errors (e.g., subtracts smaller digit from larger) can be identified, and these simple errors can be combined (e.g., 0-N=N AND stops borrowing at zero). The extent of analysis which can be carried out by computer is greater than could be done by an individual.

3. Research Design and Procedures

The procedures have been adapted to a quasi-simulation setting in which the computer acts as an errant student and an individual tries to diagnose the bug. The individual presents problems to the computer which computes according to the selected errant behavior. When the individual believes a diagnosis is achieved, the computer presents five problems which the individual is then to compute according to the errant behavior. If the responses are consistent with the errant behavior, the true error is identified by the computer so the individual can check the diagnosis. If not, the individual presents more problems to the computer to help rediagnose the errant behavior.

This simulation was used with preservice teachers (for about 1 1/2 hours) and with seventh- and eighth-grade students (free use during a school term). No further details about experimental procedures were given.

The procedure was also applied to a data base of 1325 fourth-, fifth-, and sixth-grade Nicaraguan students. These students had previously been tested on simple and complex addition and subtraction problems, but only the subtraction results were analyzed. Single bugs and pairs of bugs were identified.

4. Findings

Anecdotal data are provided for the experiment with preservice teachers. The details of the experimental results are referenced. Exposure to the procedure improved subjects' abilities to detect regular patterns of errors.

No data for the seventh- and eighth-graders are reported.

A total of 60 single bugs were identified for the Nicaraguan students. In addition, 270 pairs of bugs were identified whose "symptoms" were different from those of the single bugs and different from each other. The single and double bugs were used to categorize students according to "likely" explanations of difficulties. Although the classification was completed, there was no opportunity to follow up with the students to verify the diagnoses.

5. Interpretations

One projected use for the procedure is to determine how well a test can diagnose errors. That is, if two bugs produce the same answers for the problems on a test, then the test cannot distinguish among these as explanations for the errors. Identifying bugs uniquely is clearly a desirable trait of diagnostic tests.

Abstractor's Comments

The paper is obviously not typical of those abstracted in IME. It is basically a feasibility study of a computer procedure for categorizing students' errors according to possible explanations of the causes of those errors. As such it is quite convincing.

The major difficulty in applying the procedures outlined would seem to be the expense in time and effort to generate appropriate computer programs for each algorithm of interest; there are many algorithms for which one might want to diagnose errors. Conceptually, the ideas seem fairly straightforward. The procedure would seem to have important potential as part of real-time, interactive, computer-based tutoring systems.

I would like to see a diagnostic test developed for subtraction which would be able to diagnose uniquely the bugs identified by the authors. Of particular interest would be the number of problems required. Field-tests of the test would also be of considerable interest to many mathematics educators.

Damarin, Suzanne K. CONJUNCTIVE INTERPRETATIONS OF LOGICAL CONNECTIVES: REPLICATION OF RESULTS USING A NEW TYPE OF TASK. Journal for Research in Mathematics Education 8: 231-233; May 1977.

Abstract and comments prepared for I.M.E. by NICHOLAS A. BRANCA, San Diego State University.

1. Purpose

To examine prospective elementary school teachers' interpretations of logical connectives when they appear in single statements concerning mathematical relations.

2. Rationale

Earlier studies report contrasting findings on subjects' performance on inference tasks (O'Brien, Shapiro, and Reali, 1971; Damarin, 1977).

3. Research Design and Procedures

An eight-item test consisting of four two-question subtests on each of four connectives (conjunction, disjunction, conditional, and biconditional) was administered to 65 preservice elementary school teachers. Items were scored 1 if correct and 0 if incorrect or omitted. Means, standard deviations, and reliability estimates were computed for each subtest and the statistics and error patterns were examined. The analysis was repeated after scoring the items as if they were all conjunctions.

4. Findings

Means, Standard Deviations, and Reliabilities for 2-Item Tests

Connective	Mean	SD	Tetrachoric r
Conjunction	1.94	.52	.95
Disjunction	.60	.74	.80
Conditional	.05	.28	.65
Biconditional	.08	.37	.93

Statistics for 2-Item Tests Scored for Conjunctive Interpretation

Connective	Mean	SD	Tetrachoric r
Conjunction	1.94	.52	.95
Disjunction	.78	.85	.91
Conditional	1.71	.52	.94
Biconditional	1.80	.48	.97

5. <u>Interpretations</u>

Within the context of elementary school mathematics, preservice elementary teachers have a better understanding of the inclusive "or" than of implication. Most subjects consistently treated the conditional and biconditional as if they were conjunctions and 17 of the 65 subjects interpreted the disjunctive "or" that way. Fifteen subjects treated the disjunctive "or" in a manner consistent with the logical "inclusive or."

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<u>Abstractor's Comments</u>

The major question regarding this study is its generalizability to both subjects and content. The test items were specific to odd and even numbers and cannot be said to represent the context of elementary mathematics. The subjects were preservice elementary school teachers enrolled in the first of two required mathematics courses. Their background in logic is not described, yet the interpretation of the results depends on whether the test was administered before or after exposure to this content. If the subjects had no exposure to logic concepts (as one would assume from the report of the study), then their tendency to treat the conditional and biconditional as equivalent to the conjunction is understandable.

Erlich, Oded and Shavelson, Richard J. THE SEARCH FOR CORRELATIONS BETWEEN MEASURES OF TEACHER BEHAVIOR AND STUDENT ACHIEVEMENT: MEASUREMENT PROBLEM, CONCEPTUALIZATION PROBLEM, OR BOTH? Journal of Educational Measurement 15: 77-89; Summer 1978.

Abstract and comments prepared for I.M.E. by JOHN DOSSEY, Illinois State University.

1. Purpose

The purposes of the study were "(a) to provide data bearing on the generalizability of measures of teacher behavior over several facets of their measurement, and (b) to determine whether the lack of significant correlations between measurements of teacher behavior and student achievement is due to problems in the generalizability of their measurement, in their conceptualization, or both."

2. Rationale

With the advent of operationally defined variables in the study of teaching, several attempts have been made to find significant correlations between various measures of teacher behavior and student outcomes. The resulting low correlations indicate that teaching acts or teaching effects on student achievement are unstable for the most part (Shavelson and Dempsey-Atwood, 1976). To understand the reasons behind these low correlations, generalizability theory (Cronbach, Gleser, Nanda, and Rajarathnam, 1972) is applied to the analysis of the data from one such study. This results in a set of teacher behavior variables which can be reliably measured over occasions and raters in studies of teacher effects.

3. Research Design and Procedures

The study analyzed samples of 10 fifth-grade teachers' behavior in instructing their reading and mathematics classes. This was a reanalysis of some data which had been collected in the Beginning Teacher Evaluation Study. Each teacher was observed on three different days (occasions). On each of these days, both the 40-minute reading and mathematics classes were taped (subject matter). These video tapes were then rated by a panel of observers on a cluster of teaching behaviors (raters).

The video tapes were coded with respect to both the frequency of particular teaching behaviors and global aspects of the teaching performances. The resulting data were structured for analysis using the following five clusters of teacher behavior variables: (a) presentation of content, (b) teacher questions, (c) teacher feedback and classroom management, (d) interpersonal behavior, and (e) global ratings.

Due to the small number of teachers, raters, and occasions, the data were checked for congruence in the error square magnitude with the data from the previous Shavelson and Dempsey-Atwood study to prevent the interpretation of the spurious effects of some chance variation. The data were then analyzed by a computer program developed by the authors which split the variance into the universe score variance and three sources of error variance: the teacher-occasion interaction confounded with occasion variability, the teacher-rater interaction, and the teacher-rater-occasion interaction confounded with the rater-occasion interaction and unidentified sources of error. These error sources were labeled as the O-term, R-term, and E-term respectively.

A particular teacher behavior variable was labeled as generalizable if its relative universe score variance exceeded 0.10. This coefficient of generalizability is likened to the index of reliability in classical test theory. These coefficients were computed across the 10 teachers, 4 raters, and 3 occasions.

4. Findings

Within the cluster of teaching behaviors called presentation of content, only one of the 10 variables was judged generalizable. This was the variable called "giving an explanation." Its error variation was almost entirely explained by the E-term (89 percent). Since it was the only variable in this cluster judged generalizable, no pattern could be established for its measurement. The instability of the other variables in the cluster agreed with the earlier Shavelson and Dempsey-Atwood findings.

In the second cluster, teacher questions, three of the seven variables were judged generalizable. These variables were "asking for an answer," "asking if information was understood," and "asking for agreement." The e' or for these three variables was due to the O-term and the E-term and not to the R-term. Hence these variables can be measured reliably with a moderate number of occasions. This finding was in partial agreement with the Shavelson and Dempsey-Atwood study.

In the teacher feedback and classroom management and control cluster, the variables were divided into the subcategories of positive, neutral, and negative feedback. The analysis of the first subcategory showed that three of the positive feedback variables were generalizable (simple knowledge of results, elaborated knowledge of results, and positive physical feedback). The pattern of error variation showed an even distribution of each of the sources, indicating that these variables can be reliably measured with a minimal number of raters and occasions. In the neutral and negative feedback subcategories, the variables exceeding the 0.10 criterion did not have patterns which would allow the explanation of the conditions under which they would be generalizable. Again these findings generally agreed with those of the earlier study.

Five of the interpersonal behavior variables were judged as being generalizable. The first group, called probing behaviors, were: "directions to child to elaborate with knowledge of results," "asking for development of a response," and "direction to try again." The pattern of the sources of error variation for these measures was evenly distributed across the sources of error variation. Hence, these measures can be reliably made with a reasonable number of raters and occasions. The two variables labeled generalizable in the non-probing subcategory had the majority of their variation due to the E-term. In order to increase the generalizability of these measures, additional study would have to be made of the sources behind the magnitude of the E-term.

In the global rating cluster, several variables were judged as being generalizable. For most of these the source was the O-term and the R-term, with relatively low contributions due to the E-term. These findings suggest that the global behaviors can be reliably measured with a reasonable number of raters and occasions. These findings also agreed with the earlier study on the generalizability of the global variables.

5. Interpretations

The study of the findings concerning variables labeled as non-generalizable in studies of teacher behaviors and student outcomes in general shows a pattern of inconsistent correlations. The few cases where this is not the case need to be examined closer to see if the problem resides in the measurement procedures employed or in the conceptualization of the variables. The study of the findings showing consistent non-zero correlations in studies of teacher behavior and student outcomes tend to be on variables labeled here and in the previous study as being generalizable measures.

Before additional studies of correlations between teacher variables and student outcomes are made, the generalizability of the various measures needs to be examined with respect to the number of raters and occasions necessary to make any kind of generalizable statements. In cases of non-generalizable variables, the cause for the non-generalizability needs to be examined. Is it due to the conceptualization of the variable or the measurement process or possibly both?

Abstractor's Comments

The present study was well done. The authors carefully constructed their model and provided adequate safeguards against spurious findings. While their numbers were low, they carefully and consistently interpreted their findings in light of the model and the data. The results are important and should lead to substantial reanalysis of many mathematics education studies which have attempted to relate teaching behavior variables to student outcomes. As such, the article is a valuable contribution to the literature on the development of methods for the study of teaching and the construction of theories of classroom instruction.

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Geeslin, William E. and Shar, Albert O. AN ALTERNATIVE MODEL DESCRIBING CHILDREN'S SPATIAL PREFERENCES. Journal for Research in Mathematics Education 10: 57-68; January 1979.

Abstract and comments prepared for I.M.E. by KAREN FUSON, Northwestern University.

1. Purpose

This study examined characteristics of test items used in researching Piaget's topological-to-Euclidean hypothesis. The hypothesis proposed in this study was that performance will vary with the amount of distortion of a choice item from the standard item, rather than with the particular type of distortion (topological vs. Euclidean) regardless of its difference from the standard. A particular method of measuring the amount of distortion in geometric figures is proposed and tested.

2. Rationale

Previous research attempting to ascertain whether topological concepts are understood before Euclidean concepts has yielded contradictory results. Many test items in this literature consist of a standard and two choice items, one of which is a topological variant of the sample item and one of which is a Euclidean variant. Subjects are asked to choose which one of the choice items is most like the standard. The results are examined for a shift with age from topological choices to Euclidean choices. However, the amount of difference between the standard item and each choice item has been uncontrolled.

3. Research Design and Procedures

The standard and choice items were viewed as being composed of sets of dots (as if they were drawn on lattice-point paper). The difference between a standard and a choice item was defined as the number of points required to change the choice item to the standard. A priori probabilities for the selection of each variant were calculated by dividing the number of points required for one variant by the sum of the points required for both variants. Topological and geometric variants of each standard were selected in such a way that the a priori selection probabilities for the topological variant varied from .09 to .91 across the ten test items. Choices were predicted to match these probabilities. Four versions of the test were used, each version being composed of a different random sequence of items.

The test was individually administered to 345 children from nursery school through fourth grade. Each sample with its topological and Euclidean variant was on an individual page. Each subject was asked to point to the figure that was most like the top figure.

The correlation between the a priori selection probabilities and the actual selections was calculated. Effects of school level (age) were examined for the individual test items and for the test as a whole. For the individual test items, a likelihood ratio test was employed, with significance levels determined by the chi-square approximation to $-2 \ln \lambda$ (λ = the likelihood ratio). A one-way analysis of variance was run using the sum of each student's responses. Likelihood ratio tests were used for assessing the effects of the test version, and a standard normal test was used for examining the effects of sex.

4. Findings

The Spearman rho for the correlation between the predicted choice frequency of topological variants and the actual frequency was .97. With the exception of one item, the star, the rank order of the items on the actual frequency was the same as the predicted rank order. The frequency of choice of the topological variant was significantly different across grade levels for four items. These differences seemed to result primarily from performance by the nursery school subjects, which differed from that of the older children. Except for the anomolous item four (the star), the rank ordering of items by frequency of choice for grades 3 and 4 and for kindergarteners was identical to the predicted order, the rank ordering for second grade contained one reversal of the predicted order, that for grade 1 contained 2 reversals, and that for the nursery group contained 3 reversals. The age difference on the test scored as a whole was not significant. The chi-square analysis on item frequencies in different test versions was significant for four test items, and the ANOVA for total scores for different test versions was significant. Examination of frequencies for particular items in different versions yielded no clear explanation, either mathematical or psychological, of possible sources for these differences. The different random versions happened to permit assessment of effects of the immediately preceding item and of placement early or late in the test. Neither of these was responsible for the differences found. Nor was there any evidence of a right-left bias. There were no significant sex differences in the frequency of selection of topological variants for particular items or for overall test scores. Post-test interviews in which items were given again and reasons for choices assessed led to the identification of a small number of students at each age level who strongly favored either topological or Euclidean variants across eight or more items.

5. Interpretations

The authors interpret their findings as providing strong support for their distortion model. They conjecture that the fact that the fourth item (the star with interior lines drawn in) resulted in almost all subjects of all ages choosing the Euclidean variant (star with omitted interior lines), rather than the topological variant (a flower), was an effect of familiarity with the shape. Several general possible explanations for the age differences in choice are proposed: early developmental changes in perception, smaller sample size, less understanding of the task, minimal exposure to spatial figures. It is concluded that "the youngest children responded in a somewhat different manner," but this difference is not characterized further. The finding of significant performance differences resulting from the sequence of items in a test (the version used) was cited as further support for the idea that test items, rather than developmental differences, are responsible for the variation in performance in such tasks.

Abstractor's Comments

The authors' invention of an operational definition for the difference between (or distortion of) a choice item figure and a standard figure is an extremely useful methodological control for research with geometric figures. The evidence seems quite convincing that, with the exception of familiar figures, the frequency of choice of an item approximately matches that determined by the difference definition. This study serves as another important reminder that task variables must be controlled before we can make inferences about other sources of performance differences.

This study stemmed from a methodological concern with research in this area. A very creative way to deal with one important difficulty in this area was invented. A considerable effort then went into testing this method: a large sample was individually tested. However, the authors could have given greater consideration to their findings or of implications of their findings. The most important limitation in this regard is with respect to the application of their definition of difference to previous studies. They open their paper by suggesting that previous research has lacked an important test-item control. They conclude that their method does provide such a control. It would have been extremely informative and fairly simple for the authors to have applied their method to the figures used in the studies to which they referred in the beginning of their paper. As it is, their paper leaves the impression, overall, that previous research results on preferences in topological or of Euclidean variants of a figure reduce simply to being effects of the test items used. The extent to which those results in fact stem from the nature of the items used can now be assessed by the method proposed in this paper, but it must be used before any conclusions can be drawn. The authors' treatment of this issue seems somewhat peculiar. They state, "If the measure can be used with previous data, then this whole paper seems rather pointless. The authors were in a position not only to know whether their measure could be used with previous figures (presumably the method works with any figures), but also actually to do so.

Furthermore, the authors fail to discuss adequately the implications of their scaling procedure for assessing Piaget's hypothesis. This study has provided a way to generate test items matched for distortion from the standard. It has even provided a measure of such distortion. Studies can now be designed to examine not only items that are properly matched, but also to ascertain the effect of varying the amount of distortion (e.g., both choices far from standard, both choices close to standard). These considerations should have been explicitly discussed in the paper. Instead of such a discussion, there is only the rather strange sentence: "It may be that Piaget's assertions are valid only when variants at the same distortion level are used." Presumably, the point of their study was to develop a method of determining when variants were at the same distortion level so that Piaget's assertions could then be examined with those distortion levels controlled (i.e., equal).

A careful consideration of other aspects of the figures might have enabled the authors to account for their age and test version results. The ability of figures to be labeled might be one important factor. The existence of a label might account for the previously mentioned overwhelming choice of the star without internal lines. It might also help to explain performance on the sixth item, whose standard was a circle and whose overall choice level of the part of the circle was 10 percent higher than the predicted probability. Another characteristic of test items that might be examined is the extent to which part-whole relations were maintained or violated. On some items, missing line segments were in the interior of the figure, so that their omission left the whole the same. On other items, the omission resulted in only a part of the figure remaining. Most of the differences between the nursery school sample and the older children seemed to be on items involving these part-whole problems.

Finally, the authors' treatment of the age differences they found could have been more analytical. They had 44 subjects at the nursery school level; they might have split that sample into high and low (e.g., three- and four-year olds) age groups to pursue the age differences they did find. They might have checked their interview data for clues about sources of these differences. A considerable amount of data were generated for this study, and a large investment of time went into the collection of these individual performances. Any suggestions the authors might make as a result of each further examination of their data would not carry, of course, the same weight as a study in which characteristics were specifically manipulated, but the hypotheses provided in such an examination would have been a welcome complement to the methodological advance that is the main contribution of this paper.

Johnson, David W.; Johnson, Roger T.; and Scott, Linda. THE EFFECTS OF COOPERATIVE AND INDIVIDUALIZED INSTRUCTION ON STUDENT ATTITUDES AND ACHIEVEMENT. Journal of Social Psychology 104: 207-216; April 1978.

Abstract and comments prepared for I.M.E. by WILLIAM NIBBELINK, University of Iowa.

1. Purpose

The purpose was to compare learning outcomes for a "cooperative" situation and an "individualistic" situation, using as dependent variables: (1) attitudes toward teacher and classmates, (2) attitudes important for socialization of students into society, and (3) achievement relative to the mathematics studied. The "cooperative" situation involved students working in groups of four, seeking help from each other rather than from the teacher, with the teacher rewarding the group as a whole. The "individualistic" situation involved students working on their own, avoiding interaction with peers, securing help when needed from the teacher, with the teacher rewarding the individual.

2. Rationale

"While there is a great deal of research comparing the relative effects and correlates of cooperative and competitive goal structures, there is an absence of research comparing cooperation and individualization." Furthermore, the homogeneous setting is usually neglected for more integrated settings relative to the study of attitudes important for the socialization of children into a heterogeneous society.

3. Research Design and Procedures

Of 120 fifth and sixth graders, the 30 with the highest scores on a mathematics test were selected and then ranked on the basis of mathematics performance over the five preceding months. The first 14 even-numbered students (3 boys and 11 girls) were assigned to the individualized condition; the other 16 (9 boys and 7 girls) to the cooperative condition. Four cooperative groups were formed, each consisting of a high achiever, a low achiever, and two average achievers from the 16. All students were from upper-middle-class homes, and, as a group, the 30 were above average academically.

All students met at the same location under the same teacher for 60 minutes of mathematics a day for 50 days. The mathematical content was structured for use as an individualized program and was new content to the students. Each day students in the cooperative condition moved their desks together to form their groups of four, and students in the individualized conditions moved their desks to isolate themselves from other children. Data were collected during the 50 days, at the end of the period by a posttest, an two months later by a retention test.

4. Findings

"The results indicate that cooperative learning promoted more positive attitudes towards heterogeneity among peers, higher self-esteem, more positive attitudes toward the teacher, fellow cooperators, and conflict; more internal locus of control; and higher daily achievement."

On both the mathematics posttest and the mathematics retention test, the students in the individualized condition did better when the tests were taken individually.

5. Interpretations

Clearly, the students involved in the study constituted a homogeneous group, being white, upper-middle-class, high-achieving children. "In such homogeneous settings teachers can increase student valuing of ethnic, sex role, and cultural heterogeneity among peers through structuring, learning situations cooperatively." However, "generalization of results is limited by the specific math materials used, the nature of the students participating in the study, and the size of the sample."

Abstractor's Comments

Although it is fashionable to use the student as the unit of sampling with studies such as this, it is questionable. The teacher was clearly part of both treatments, and not equally part of both. The authors reported that many in the individualized condition mentioned "not liking to wait for the teacher's help" and being concerned "that they would appear dumb." Such feelings about seeking help could have been determined in part by the teacher, which could, in turn, have implications for any measure of attitude administered in that setting.

Both groups came from an individualized condition into the conditions set up for the study. Thus, the individualized condition was a continuation of a condition while the cooperative condition represented a change. The problem of differences being in part a reaction to change exists.

Administering both conditions in the same room at the same time admits its own batch of problems. If an analogy is permitted, hamburgers taste different when everyone is having hamburgers than when half the folks are enjoying steak. The authors suggest that the children who worked in groups had more positive attitudes toward conflict than those who sat quietly alone during whatever conflicts took place in that room. This may suggest simply that argument is more valued by those who argue than by those who are forced to hear it. Also, the less positive attitudes toward the teacher by those who sat alone may be a reaction to being deprived of visiting others by that person, for 50 days, while other students were encouraged to visit. Briefly, there is a considerable risk with this study that the measures of attitude yielded results peculiar to the very atypical experimental conditions.

The study should be viewed as a pilot study which presents some interesting questions and offers suggestions for treatment conditions and dependent variables. General statements about the relative merits of the "individualistic" situation and the "cooperative" situation are not warranted.

Kutz, Ronald. AN ANALYSIS OF THE USE OF MATH MANIPULATIVE MATERIALS IN NORTH DAKOTA. Grand Forks, North Dakota: Bureau of Educational Research and Services, August 1977.

Abstract and comments prepared for I.M.E. by CLYDE A. WILES, Indiana University Northwest.

1. Purpose

This survey study was to provide information "needed to plan better pre and inservice mathematics programs" for Elementary School Teachers. The questions to be addressed were identified as follows (page 2):

1. What mathematics manipulative materials are currently being used in North Dakota elementary school classrooms, and to what extent?

2. Who is using them?

2. Rationale

The basic rationale derives from two beliefs. First, that physical interactions of children with the environment produce mental growth and understanding. And second, that the use of mathematics manipulative materials (mmm) has remained fairly limited in the schools. The reason for this study was thus "the perceived discrepancy between theoretical need for manipulative experiences and common teaching practices focusing on the textbooks." The goal was to discover what factors affect the use of mmm in the classroom and to design responses at the preservice or in-service level to effect an "increased intensity and diversity in use of mmm."

3. Research Design and Procedures

The basic design was a self-report questionnaire survey of teacher characteristics, practices, and attitudes concerning mmm, a tabulation of these data, and a series of correlations between a measure of "intensity and diversity" of the use of mmm and the other factors surveyed. Population: The population sample was all the elementary teachers of North Dakota elementary schools with level 1 accreditation (approximately 1000) and enough randomly selected schools with level 2 and 3 accreditation to provide a pool of 500 teachers from each of these levels. A total of 2100 questionnaires were distributed to a total of 147 schools.

Questionnaire: The questionnaire consisted of 21 items. The content of the questionnaire was reviewed by seven people not directly associated with the surveys. The questionnaires were distributed to building principals of the selected schools. Principals were requested to give them to their teachers, and then collect and return the completed forms. No attempt was made to ascertain which schools or teachers returned completed forms, nor, for that matter, the proportion of responses within a given building. The total return was 989 (47 percent) questionnaires from 116 (79 percent) of the schools.

Data Analysis: A number of tables presented the responses by grade level taught as well as by total. In addition to reporting the data by item and category, another variable was constructed. In brief, if a teacher checked that mmm were used "a little," a score of 1 or 2 was determined on the basis of the number of different kinds of mmm that were used; if a teacher checked that mmm were used "quite a bit", a score of 3 or 4 was determined; and if a teacher checked "very extensively", a score of 5 or 6 was determined on the same basis as before. This constructed variable was correlated with other measures.

While several measures of correlation were computed and reported, the Pearson r is the statistic generally used in inferring relationships among the measures.

4. Findings

The frequencies of the various items being used, in the order of most to least were as follows: metric materials (72 percent), counting chips (45 percent), bundles of sticks (44 percent), geometric construction materials (39 percent), fraction discs (37 percent), tangrams (28 percent), geoboards (23 percent), calculators (22 percent), attribute materials (21 percent), and Cuisenaire rods (20 percent). Math balances, fraction bars, "other," and Multibase Arithmetic Bars were all less than 20 percent.

As to the extent of use, 35 percent of the teachers reported the use of 2 or 3 different types of materials, and 29 percent reported the use of 4 or 5. Forty-eight percent (48%) reported "little" overall use and only 7 percent reported "extensive" use.

Many correlations with the constructed intensity-diversity measure were significant beyond the .0001 level. The large sample size produced significance for r values as low as .11.

The following correlations were commented upon by the author.

With respect to source of materials: Teachers who own (.33), construct (.25), or request (.29) materials are more likely to use them than those who do not. But teachers with the greatest variety of available materials (.44) are the most likely to use them.

With respect to source of knowledge about materials: The most effective source was professional reading (.25), followed by in-service (.19), graduate courses (.17), and undergraduate courses (.18). The most significant factor, though, was the total number of sources (.39).

With respect to teacher belief about the effectiveness of mmm: There was a much higher relationship between the use of mmm and the belief that fast (.23) or average (.24) learners benefit from their use, than with the belief that slow learners do (.08). However, 98 percent of the teachers believed slow learners benefited from the use of manipulatives, rendering this factor impotent in distinguishing anything.

Finally, use of reference books (r = .26) and knowledge of manipulative use (.25) were cited as showing notable strengths.

5. Interpretations

Conclusions (paraphrased):

1. There are many materials in the schools.
2. Extent of use is not particularly high.
3. The relationship between mmm use and learning centers is to be expected. It is not clear if the desire to use mmm produces learning centers or conversely.

4. The lack of relationship between use of mmm and special education settings was disappointing.

5. Personal involvement by the teachers in obtaining mmm and the apparent professionalism of teachers seem to produce the best prediction of mmm use.

6. The relative high belief in the value of mmm together with the relative low usage indicate a large market potential. However, teachers seemed ill-informed about the true cost of materials.

Recommendations (paraphrased):

1. The fastest way to increase intensity of use of mmm is to increase the use of those already in the classroom.

2. Since personal involvement in procurement of mmm is related to use, classes for teachers at every level should include mmm packages.

3. Since personal reading is related to use of mmm, efforts should be directed to increase the use of journals such as the *Arithmetic Teacher*.

4. Support should be provided for the development of learning centers.

5. Further research that better defines the construct "extent of use" of mmm should be undertaken.

Abstractor's Comments

This status study was well done and does provide information that can aid in the planning of in-service and preservice courses. The author was obviously aware of the difficulties associated with such studies and took steps to avoid them. The limitations of the various procedures were recognized for the most part, and the implications appropriately restricted. But the study is not without its problems.

1. There is no justification presented for the use of r when it seems that the Spearman Rho would have been the preferred statistic to measure relationships between what are at most ranked

measures. The Rho is clearly more conservative and the sample size provided an overwhelming amount of statistical power.

2. The probable effect of the special sample and the probable effect of the principals' input to the teachers was not addressed adequately. While it seems likely that such effects would tend to inflate the apparent usage of mmm, how this might affect the other variables and the correlations is not at all clear.

3. While implicit in the title and setting, there is some question about the generalizability of relationships observed in North Dakota to other places such as the Chicago area, for an extreme example.

4. The selection of manipulatives the teachers were to choose from included calculators and metric measures. Traditional measuring devices such as rulers, pints, quarts, etc., were not included. Furthermore, no teacher was reported to have listed these in the "other" category. While this is not a criticism of the study, it is curious.

The author calls for additional research using more clearly defined variables relating to the extent of use of mmm. This seems necessary if we are to respond adequately to the in-service needs of teachers. I would add that such investigations in other settings are also in order. Of course, status findings of relationship do not indicate causation. While the data do suggest valid directions for in-service work, they do not demonstrate that the systematic inclusion of such things as manipulative packages and subscriptions to the Arithmetic Teacher would actually produce the desired effect. Even more interesting than the mere use of mmm in the classroom, though, is the probable effective use of such materials.

In summary, it is this reviewer's opinion that, while the recommendations as presented are supported by the data, care must be taken to provide adequate theoretical frameworks for the use of mmm in the classroom. The experimental evidence that was the starting point for this survey involves the use of mmm in carefully structured ways. I'm not aware of a body of research that indicates the mere inclusion of mmm in the curriculum is associated with or productive of efficient learning. However, the study was well done, presents interesting data, and is well worth the consideration of those who are involved with the training of teachers of elementary school mathematics.

Lesh, Richard and Mierkiewicz, Diane (Editors). RECENT RESEARCH CONCERNING THE DEVELOPMENT OF SPATIAL AND GEOMETRIC CONCEPTS. Columbus, Ohio: ERIC Clearinghouse for Science, Mathematics, and Environmental Education, May 1978. ERIC: ED 159 062.

Abstract and comments prepared for I.M.E. by MICHAEL MITCHELMORE, University of the West Indies, Jamaica.

1. Content

This book contains eleven research papers and three review articles concerning the learning of spatial and geometric concepts from infancy to college age, together with an introduction by the editors and a summary by Arthur Coxford. All the studies were completed in the 1975-76 academic year by members of the Space and Geometry Working Group of the Georgia Center for the Study of Learning and Teaching Mathematics.

2. Background

The Space and Geometry Working Group is one of nine working groups set up after a series of research workshops organized in 1975 at the University of Georgia. The group meets several times a year to identify important issues, formulate researchable questions, plan research projects, and communicate results. It is believed that such collaboration will lead to studies which are "more basic, more to the heart of the matter, and consequently more important," that more complex issues can be investigated, and that the optimal time for communication between researchers is while projects are being planned.

3. Brief Abstracts

A. Studies concerning pre-operational concepts

In the first paper, Lesh and Mierkiewicz review the distinction between perceptual and conceptual processes, and relate both of these to the construction of images. They consider how a percept is extracted from the visual array, emphasizing the influence of the observer's representational system, and briefly discuss visual scanning, the role of perceptual activity, and the interpretation of ambiguous figures. Finally, the term "image" is defined, characteristics of "good" images as multifaceted symbols are described, and the relation of imagery to perception and conception is discussed.

Lesh is also the author of the second paper. He classified 169 kindergarten children into five operational ability levels in seriation. All children were shown a triangular array of six poker-chips which they were then asked to reproduce or recognize one week and six months later. Scores on the criteria tasks were closely associated with seriation level. There was a significant improvement in scores from one week to six months, but it was confined to the second and third lowest seriation levels. The author suggests that the encoding and decoding phases of memory are both influenced by the subject's operational ability, and that those whose scores improved had encoded the array correctly, were unable to decode it at one week, but had developed sufficiently to be able to decode it more accurately at six months.

Fuson and Murray tested 96 children aged two to six years from two backgrounds on the haptic recognition, construction (using large or small sticks) and drawing of small (palm-size) models of a circle, triangle, square, and diamond. The hypothesized orders of difficulty for haptic-construction-drawing and circle-square-triangle-diamond were strongly confirmed. Almost all children haptically identified all figures by 3.6 years. Over two-thirds constructed the square by age 4.0, the triangle by age 5.0, and the diamond by 6.0. However, even at 6.0 years, less than one-third were able to draw any shape other than the circle accurately. Implications and secondary results are discussed extensively.

Musick gave 142 children aged 3½ to 9 years a variety of movement tasks intended to highlight the symmetry of distance measurement. In some tasks, the child walked, ran, or jumped across and back between two points, sometimes carrying a load; in others, the child or the investigator moved a doll. Children were asked to judge and justify the equality of the two distances. There was no significant difference in performance between child and doll tasks, or between participant or observer tasks; however, performance following walking was significantly superior to that following running and jumping. The results are interpreted in relation to the role of activity in spatial learning.

In the final paper in this section, Weinzweig summarizes the Klein-Erlanger Program, reviewing basic concepts and results pertaining to the groups of motions and similarities, the affine and projective groups, and the group of homeomorphisms. He then considers some aspects of research on space and geometry from the Klein viewpoint.

B. <u>Studies concerning transitional stages from concrete to formal operations</u>

(1) Four studies in this section concern rigid motions. Thomas gave 50 students from Grades 1, 3, 6, 9, and 11 three tasks. In the first, students indicated whether lengths on the image were equal to corresponding lengths on the original figure; in the second, they located points on an image corresponding to a given point on the original figure; and in the third, they predicted the images of given letters of the alphabet. All three tasks involved a variety of slides, flips, and turns. On the first task, most students seemed to believe length was invariant under flips and turns, but classical non-conservers of length tended to believe length changed under slides. In the second task, while most first graders chose the correct side even though the distance was often wrong, many third graders chose the nearest vertex. Performance on the third task was strongly influenced by the symmetry of the letter used; Grade 3 performed significantly lower than Grades 6, 9, and 11.

Schultz asked 270 children aged six to ten years to predict the outcome of various slides, flips, and turns demonstrated by the investigator. Tasks varied according to the size and meaningfulness of the figure and the size and direction of the displacement required by the transformation. Slides were easiest, horizontal displacements were easier than diagonal ones, moderate displacements were easier than short (overlapping) or long ones, meaningful figures were generally easier than non-meaningful ones, and large figures were easier than small ones. The most frequent error was in the orientation of the image.

Kidder specifically investigated the concept of length invariance. Sixty children from grades 2, 3, and 4 were first tested for classical length conservation; they were then asked to predict the image of a 125 mm long stick under 12 given transformations; the subject had first to choose a stick of the correct length from five given. The correct length was chosen 9 or more times by only 7 of the 31 conservers and 6 of the 29 non-conservers. The author suggests that the subjects' conception of geometry was at an affine stage, so that they made images which were like but not congruent to the original figures.

Perham trained a random half of a sample of 72 first-grade students on slides, flips, and turns in 11 half-hour sessions, using tracing paper, geoboards and free-drawing activities in lecture-discussion and small group modes. The posttest consisted of 28 multiple-choice items and 52 drawing items; a transfer test of 23 items selected from commercial spatial ability tests was also given. Training appeared to be successful in teaching understanding of horizontal and vertical slides and flips and was somewhat effective for turns, but there was no significant improvement in the understanding of diagonal slides or flips. Transfer to spatial ability was restricted to items on perspective.

(2) Two papers deal with "middle geometrics"--projective, affine, and similarity geometrics.

Fuson presents a critical analysis of the relevant experiments from Piaget's Space and Geometry books, summarizes the results, and suggests several areas where further research would be valuable.

Martin reports a study on the conception of ratio. Forty children from Grades 3, 4, and 5 were shown a number of models, each consisting of a rod with a fixed bead on it dividing the rod in the ratio 5:2. Twelve tasks tested the child's ability to preserve this ratio on rods of different lengths. The subject selected the appropriate model in five tasks, placed a bead on a given rod in four, and drew a rod and bead in three. A number of "helper sticks," of the same length as the test models but marked off in seven congruent subdivisions, were available to the subjects. Performance among third and fourth graders was very poor. Among fifth graders, 7 out of 20 made correct responses to all nine of the concrete tasks, and 9 made correct drawings. The "helper sticks" were used by 30 percent, 50 percent, and 95 percent of the children from Grades 3, 4, and 5, respectively. The most common error was to conserve the distance of the bead from one end of the rod.

C. Studies concerning older subjects or formal operational concepts

Mayer and Johnson report three studies. The first study involved 120 children from preschool to third grade and confirmed the developmental order slide-flip-turn in contrast to the logical order in which flips are more primitive. The second study, with 84 subjects from Grade 5 to college, found that locating a series of four colored balls in a 4 x 4 x 4 cubical array was empirically more difficult than the logically more difficult tasks of locating it in a 7 x 7 square array or six 4 x 4 square arrays. In the third study, 120 fourth graders completed a self-instructional unit which dealt with the definitions and elementary properties of reflections and rotations. The results showed that subjects receiving advanced organizers scored significantly higher than those receiving post organizers or no organizers. The authors argue that both psychological and mathematical structures must be taken into account when designing curricula.

Dietz and Barnett gave Piaget's water-level and plumb-line tasks as a group test to 236 elementary education majors. Only 43 percent drew the water level horizontally, although 82 percent drew the plumb-line correctly. A random sample of 55 subjects was interviewed. None of the 29 who had given incorrect responses to the water-level task could justify their construction; 22 of them gave different responses when asked to show the level on a tilted (empty) bottle. Ten of the 29 could not see that the surface of the water remained parallel to the table even when it was demonstrated with a half-full bottle.

In the final research study, Fisher investigated the influence of instructional materials on conceptual learning. Subjects were 168 students from Grades 6 and 9 and college. At each level, students were randomly assigned to one of three concepts ("altitude of a triangle," "angle of incidence," and "complete 4-points") and then to one of four instructional formats. In Format A, all figures were upright, in Format B, all were oblique; Format C began with upright figures and changed to oblique figures; Format D reversed this order. The posttest contained both upright and oblique figures. Scores on the upright figures were generally higher than scores on the oblique figures, but there was no significant difference between the four instructional formats.

Abstractor's Comments

This book is a gold mine of information and ideas, and is a must for everyone at the forefront of research on spatial and geometrical concepts as well as those who follow a little way behind. The standard of the papers is uniformly high and demonstrates clear thinking on the purposes and rationale of each research project, careful analysis and interpretation, and a refreshing lack of inhibition as regards conjecture and self-criticism. It is a pity that space does not allow the eleven research studies separate abstracts in I.M.E.

Some criticisms can, of course, be made. Weinzweig's paper is long and heavily mathematical without any apparent relation to the research reported in other papers. Dietz and Barnett's study is a replication of Rebelsky (1964), to which no reference is made. Other comments would be minor and out of place.

It appears that the idea of forming collaborative working groups is paying dividends. Several of the papers in this collection might have been regarded as narrow and trivial had they appeared in isolation, but taken in context their value is obvious. The publication format also has the advantage of allowing authors space to evaluate their own procedures and interpret their results much more deeply than is generally possible in a journal article.

One goal of the Working Group is to avoid the situation where contacts between researchers are delayed until "a year or two after projects have been completed and reports finally appear in journals or at conferences." This is all very well for the in-group, but this book covers research carried out in 1975-76, was published in 1978, and is now being reviewed in 1979--no better than the standard schedule. The strength of this publication lies in the collation of so many results and insights and the many perceptive suggestions for further research; it is tantalizing for an outsider to wonder how much has been done between 1976 and 1979 which he knows very little about. One hopes that the Working Group, having asked individuals who are interested in cooperating in the research effort to make contact with one of its leaders (Lesh at Northwestern, Weinzweig at Illinois, or Steffe at Georgia), are able to handle the volume of applications.

Reference

Rebelsky, F. Adult Perception of the Horizontal. Perceptual and Motor Skills 19: 371-374; 1964.

Michaels, James W. EFFECTS OF DIFFERENTIAL REWARDING AND SEX ON MATH PERFORMANCE. Journal of Educational Psychology 70: 565-573; August 1978.

Abstract and comments prepared for I.M.E. by BOYD D. HOLTAN, West Virginia University.

1. Purpose

The purpose of the study was to investigate the effect of reward for reinforcement or competition and reward for individuals or groups upon achievement of the solutions of three-step mathematics problems. Sex differences were also investigated under these conditions. The hypotheses of the study were:

Hypothesis 1: Individual performance will vary directly with differential group rewarding, being higher in the differential group rewarding conditions than in the nondifferential group rewarding conditions.

Hypothesis 2: Individual performance will vary directly with differential rewarding within groups, being higher as the individual receives greater reward in the group.

Hypothesis 3: The mathematics performance of males will be higher than that of females.

Hypothesis 4: The mathematics performance of females will show greater responsiveness to differential group and individual rewarding than will that of males.

2. Rationale

The study investigated four basic reward structures: individual and group reward contingencies and individual and group competition. The first two are associated with the reinforcement approach to structuring rewards while the latter two are associated with the competition-cooperation approach. Under reward contingencies, the probability of magnitude of rewards for one unit is unrelated to other units but determined by previously ascertained criterion. Under competition, the performance of each unit is evaluated relative to the performance of other units and rewards allocated accordingly. Either of these two reward strategies may be based upon group performance or upon performance of an individual within a group.

Previous studies are mixed as to the effectiveness of individual or group reward contingencies and competition (Michaels, 1977). The literature on behavior modification documents effectiveness of both individual and group reward contingencies in strengthening performance in schools. Studies of reinforcement and competition have, for the most part, been separate, but this study is designed to compare both in the context of group and individual rewards.

Based upon reviews such as Fennema and Sherman (1977) of academically related sex differences, males were expected to outperform females on the mathematics task. It was also expected that females would perform better with individual rewarding.

3. Research Design and Procedures

Two levels of differential group rewarding were paired with three levels of differential individual rewarding within groups to form six reward structures used as treatments. The reward was an amount of money to be paid to each subject according to the reward structure assigned. The two levels of group rewarding were:

nondifferential: each group was paid the same regardless of group performance.

differential: each group was paid in direct proportion to group performance (like group piecework rate.)

The three levels of differential rewarding of individuals within groups were:

nondifferential: each group member was paid the same.

moderate differential: group members were paid in proportion to their relative performance in the group--if one member contributed 60 percent of the total group performance, he or she got 60 percent of the group pay.

high differential: the group member with the highest performance got 75 percent of that total group pay.

Table 1. Group and Individual Reward Conditions

	Individual Rewarding Within Groups		
	Non differential (Individual group members all paid the same)	Moderate differential (Group members paid according to their group contribution)	High differential (top group member paid 75% of group reward)
Non differential group rewarding (Groups paid identically)	Non differential group and individual rewarding	Moderate individual competition	High individual competition
Differential group rewarding (Group paid relative to performance of other groups)	Group reward (Group piece-rate)	Individual reward, group competition	Mixed individual competition and group differential rewards

Table 1 indicates the six treatment groups. Subjects were volunteers recruited from summer school classrooms in a state university who were told that they had an opportunity to earn a variable amount of money ($4.00 to $12.00) for one hour's participation in a pay system. Under constraints of same sex and time availability, subjects were randomly assigned to dyads which were randomly assigned to reward structure treatments. The 144 subjects were assigned to the six treatment cells, 12 males and 12 females in each.

Each subject was given 11 performance-pay trials. Each trial consisted of the subject working for two minutes on a different set of 32 three-step mathematics problems, having them scored and being paid according to the treatment reward cell assignment. The non differential group members each received $1.20 for each trial, while those in the differential groups received $.04 for each correct problem to be paid to members accordingly.

The performance measure was the number of problems completed correctly. The results were analyzed with a 2x2x3 factorial analysis of variance design for effects due to sex, differential group rewarding, and differential individual rewarding systems.

4. Findings

The results of the ANOVA indicate that there were differences between the sexes and between individual rewards, but not between groups.

There was an interaction effect between sex and group rewarding indicated; therefore, separate 2x3 ANOVA tests were run for males and females.

Hypothesis 1 was supported only for females. Mathematics performance of females varied directly with differential group rewarding, but the performance of males was higher when group rewards were fixed.

Hypothesis 2 was supported for females but not for males. Females performed better with individual rewarding within the group than males. However, the females with moderate individual rewarding within groups performed higher than subjects in the high differential and the non differential groups.

Hypothesis 3 was supported as higher mathematics performances were found for males than females.

Hypothesis 4 was supported as the performance of females was more responsive to differential group and individual rewarding than males.

5. Interpretations

Only the mathematics performance of females responded to both group and individual rewarding. The performance of males did not respond directly to either form of differential rewarding. Thus, immediate performance feedback after each trial apparently provided a stronger stimulus for spontaneous competition for males, producing higher performance regardless of how group pay was allocated between performers.

The researcher suggested that these findings may have implications for reducing the sex gap in mathematics performance by operationalizing more explicit differential group or individual rewarding with classrooms. He also suggested that the apparent spontaneous competition effect for males should be tested further.

Abstractor's Comments

Although the study does not provide definitive information about the uses of differential rewarding and competition in general, it appears to indicate that the sexes perform mathematics problems differently in response to various reward strategies. With the current concern for increasing the success of females in mathematics situations, this study suggests use of group rewards and moderate within group rewards for females.

The report indicated little about the task other than that the subjects were given sets of three-step mathematics problems to work. There is the possibility that the type of problems or problem formats may have had different effects on the groups.

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Abstract and comments prepared for I.M.E. by JAMES M. SHERRILL, University of British Columbia.

1. Purpose

1. To develop tests that are simple to devise and administer that "make the concept involved in each item as clear and straightforward as possible and ... invite the pupil to give the correct term naming it."

2. To use the tests to estimate the "understanding or lack of understanding of some of these words [selected mathematical terms]".

2. Rationale

In Lewis R. Aiken's article, "Language Factors in Learning Mathematics" (Aiken, 1972), he states that "It is generally recognized that not only do linguistic abilities affect performance in mathematics but that mathematics itself is a specialized language" (p. 359).

"In mathematics, as in most other school subjects, key concepts are referred to by specialized terms such as 'parallel,' 'factor,' or 'rotation'." In earlier research (Otterburn and Nicholson, 1976), the author had gained "an indication of the extent of the difficulty that pupils ... have with some of the words which are used commonly in school mathematics ...".

3. Research Design and Procedures

The article reports the results of two investigations.

Investigation I: An 18-item mathematics test was completed by 185 pupils being entered for CSE Mathematics [Certificate for Secondary Education Mathematics—"broadly the middle 50 percent of the whole ability range"]. Each item consisted of a problem concerning one of the 18 selected terms from the CSE Mathematics syllabus. For example, for the term "multiple" the item was "Give one example of a multiple of 30."; for the term "parallelogram" the item was "What is true about the sides of a parallelogram?" The responses were categorized as

Correct, Blank, or Confused. The results were organized according to percent of responses judged as Correct.

Investigation II: A 24-item test was completed by 46 students using the Northern Ireland CSE Syllabus A Maths. The test was the completion items in Mathematics Test L2. The results of 6 of the 24 items were not analyzed since the terms were not on the syllabus for the Syllabus A mathematics course, which is "more or less traditional mathematics." An example of a test item follows: "The numbers 60, 90, 120, 150 are all said to be ______ of 30." The responses were categorized as being Acceptable, Neutral, Blank, or Confused. The results were grouped according to the number of responses judged Acceptable. The results were based on a pilot study in one school.

4. Findings

Investigation I: One-third of the terms were "well-understood" (% Correct $\geq$ 81.5 percent); one-third were reasonably understood, but the students showed increasing confusion (55 percent $\leq$ % Correct $\geq$ 75.5 percent); and one-third were poorly understood (% Correct $\leq$ 41 percent).

Investigation II: Half of the terms were understood (# Acceptable $\geq$ 22) and half were poorly understood (#Acceptable $\leq$ 18).

5. Interpretations

"These pupils who enter for CSE mathematics have significant difficulties in understanding some of the mathematical terms in common use."

For each investigation "... any teacher can devise his own test ... to diagnose the availability or otherwise to pupils of acceptable linguistic terms for mathematical concepts."

Abstractor's Comments

The author mentions several times in the three-page article that "The intention was to discover whether a pupil understood the term rather than to assess his mathematical ability" or whether the pupil "appears to understand the concept." The object was to see if the pupil could "give the correct term naming it [the concept]."

The results of both investigations were completely dependent on the terms selected, the point in the course when the testing was implemented, and how successful the test developers were in making " ... each item as clear and straightforward as possible ...". I'm not convinced that a student who puts the term "square" in the following blank is all that confused: "A four-sided figure whose sides are all of equal length is called a _________."

The author seemed to want the results to divide the list of terms into equal-sized groups. Both lists appear to be divided based on the number of items in each group rather than on performance as stated. In Investigation II the difference in performance level between the lowest ranked term in the top group and the highest ranked term in the bottom group is 9 percent. If he had divided the list three items higher, the difference would have been 20 percent.

The author chose to stay at a superficial level for his study of language factors (i.e., choosing the correct term with no consideration of concept understanding). The investigations add little or no information to the area discussed in Aiken's article (Aiken, 1972) which was part of the stated rationale for the studies.

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Abstract and comments prepared for I.M.E. by RANDALL I. CHARLES, Wood County Public Schools, West Virginia.

1. Purpose

The purposes were "(1) to evaluate the use of Piaget's Displacement Volume task as an indicator of early formal reasoning and (2) to develop a valid paper-and-pencil task to measure the same reasoning without the time-consuming limitations of individual interviews."

2. Rationale

In the opinion of the authors, many of the concepts in science and mathematics textbooks at grade 4 and above can only be understood by those who are capable of "formal reasoning." One study suggests that Ss capable of demonstrating formal reasoning ability on Piagetian tasks can understand "formal concepts in science." The development of a valid paper-and-pencil task to assess formal reasoning would enable a teacher in a brief period of time to diagnose students' reasoning and subsequently select course content.

3. Research Design and Procedures

A sample of 586 seventh- through twelfth-grade Ss was randomly selected from across the State of Oklahoma and used to evaluate the Displacement Volume task. Each Ss was individually administered three tasks:

(1) Displacement Volume. Ss were shown two identical glass containers partially filled with equal amounts of water. Next, Ss were handed two metal cylinders of equal volume but different masses. The experimenter lowered the lighter cylinder into the glass container and the rise in water level was noted. Ss were asked to predict the rise in water level when the heavier cylinder is placed in the container.

Ss who made the correct prediction were said to be capable of formal reasoning. If an incorrect prediction was made, the heavier cylinder was placed in the container. Ss who now correctly explained the displacement were rated late concrete and those who did not correctly explain the displacement were rated as early concrete.

(2) Sinking and Floating. Ss were given one large, heavy block and one small, light block and asked whether they would sink or float. After justifying their classification, Ss experimented with the blocks and were then asked to suggest a rule governing the sinking or floating of blocks. Ss who successfully related observations to a general rule were categorized as formal operational.

(3) Pendulum. The procedures used for administering this task are not discussed.

A sample of 118 eleventh-grade Ss, 46 sixth-grade Ss, and 25 fifth-grade Ss was used to validate a paper-and-pencil instrument developed by the investigators. Groups of Ss were administered a four-item Displacement Volume paper-and-pencil test. After completing the test, Ss were individually interviewed on the Displacement Volume task described above. Item 1 on the paper-and-pencil test showed the S two identical graduated cylinders partially filled with water and two congruent circles representing glass and steel marbles. A discussion comparing the objects was provided. Item 2 showed one cylinder with the glass marble submerged and a higher water level. Cylinder 2 was identical to that shown in item 1. Item 3 asked the S to describe the results when the steel ball was placed in cylinder 2, and item 4 asked for a justification of the response to item 3. Responses on the paper-and-pencil task were rated as evidence of formal or concrete reasoning.

4. Findings

The findings for both parts of the study were reported by grade level and for the total group. The authors' discussion of the findings was limited to the total group.

It is difficult to believe that this study was the first to investigate these interview tasks.

There are several other serious weaknesses and questions related to this study which severely limit the value of the research reported:

(1) The terms "formal reasoning," "early formal reasoning," and "fully formal reasoning" are not clearly defined.

(2) The samples of students are not adequately described. One sample "was drawn from a school with students of average I.Q. = 120." What about the other students? Why were grades 7 through 12 used? Why weren't the same grades used in both parts of the study? Why weren't the findings across grade levels discussed?

(3) Were the interview tasks presented in the same sequence to all Ss? Does performance on one task affect performance on another? The paper-and-pencil test was given to all Ss immediately before the individual interview task. Does the time between tasks and/or the order of presentation affect responses?

(4) The readability of the paper-and-pencil test was piloted with eleventh- and twelfth-grade Ss and college freshmen. In the final study, the test was administered to fifth- and sixth-grade Ss.

(5) The authors suggest that all three interview tasks can provide evidence of formal reasoning. If this is true, one would expect consistent responses across tasks for an individual S. Did this happen?

Russac, R. J. THE RELATION BETWEEN TWO STRATEGIES OF CARDINAL NUMBER: CORRESPONDENCE AND COUNTING. Child Development 49: 728-735; September 1978.

Abstract and comments prepared for I.M.E. by MARKITA PRICE GULLIVER, Stephens College, Columbia, MO.

1. Purpose

The study was designed to investigate "acquisitional relations between two strategies of cardinal number: correspondence and counting" (p. 728). The investigator compared the strategies of kindergarten, first-, and second-grade children using equivalent and nonequivalent sets.

2. Rationale

Piaget [and Szeminska](1965), using the conservation of number paradigm, concluded that "conservation ability requires both cardinal and ordinal understanding" (p. 729). Brainerd (1973), using slightly different tasks, found that ordinal concepts developed earlier than cardinal concepts. While Piaget apparently allowed his subjects to count, Brainerd did not. Piaget assumed that the counting had little meaning unless supported by cognitive structures demonstrated by the ability to conserve number. On the other hand, Gelman (1972, 1975) argued that cardinal ability is explained by assuming a counting strategy in children ages 3 to 6. However, Gelman used sets of two or three elements. For sets, this small perceived cardinality or direct discrimination of cardinality becomes a confounding factor. Thus, the present study used sets of seven to ten elements.

During the standard conservation-of-number tasks, critics note that children sometimes attend to number-irrelevant cues. The children in the present study had a "collinear correspondence" task in which they paired elements from two sets into a single linear array. This eliminated problems involving "static" configurations such as those in Brainerd's study (1973) and the problems of the standard conservation-of-number tasks. However, the latter task was included in the present study to compare to the "collinear correspondence" task.

3. Research Design and Procedures

The design was 2 (correspondence vs. counting) x 2 (equivalency vs. nonequivalency). Nonequivalency was examined with respect to both asymmetric and transitive relations. Ten boys and ten girls were sampled from each grade level: kindergarten, first, and second. Half the kindergarten children were from private school; all others were from public schools in the Phoenix area.

Materials consisted of 15 blue and 7 red poker chips, a plastic container and 4 cardboard strips with 7, 8, 9, or 10 centered, equally spaced red dots. The dots and the spaces between them were the size of the chips.

Children were individually given five randomly sequenced tasks. Prior to the tasks the kindergarten children were asked to count to 10 and to identify red and blue. The tasks and scoring were as follows:

a. Equivalence by counting (4 points total). Child counted an equivalent number of blue chips into the container when shown red dots on cardboard strip.

b. Equivalence by correspondence (4 points total). Child placed equivalent number of blue chips next to, but not on top of, red dots on strip. Any sign of counting was interrupted.

Both equivalency tasks used a random sequence of the four cardboard strips.

c. Nonequivalence by counting (3 points total). Child counted 7 red dots on cardboard strip and 8 blue chips and responded to question, "Are there more red dots, or more blue chips, or as many of both?" The strip was then placed face down. The task was repeated with the 9 red dot strip and 8 blue chips. With both strips face down and the blue chips removed, child responded to transitive question about the strips. Child had to remember the number of red dots on each strip and answer which strip had more.

d. Nonequivalence by correspondence (3 points total). Child placed 8 blue chips on the 7 red dot strip next to dots and answered same question as in task 3. Again the strip was placed face down. The task was repeated with the 9 red dot strip and 8 blue chips. With both strips face down and chips removed, child responded to question of which set had more or were there as many of both.

e. Conservation of number (2 points total). Child watched as 7 red chips were placed in a row. Child made an equal row of blue chips (1 point). One row was spatially rearranged and child was asked, "Are there more red chips, or blue chips, or as many in both places?" Procedure was repeated with the other row being rearranged. Child had to explain responses (1 point).

Scores on all eight measures (equivalence by counting, equivalence by correspondence, asymmetry by counting, asymmetry by correspondence, transitivity by counting, transitivity by correspondence, conservation of number, and pretransformation by correspondence) were made dichotomous. A child passed the equivalence and asymmetric tasks if correct on all trials. The other four tasks were already dichotomous.

4. Findings

Scores were converted to proportion correct on the eight measures. Means and standard deviations were reported for each grade level. Preliminary analysis showed no effect for sex or sex by grade interaction, and these factors were therefore ignored on subsequent analyses. "Kindergarten children differed significantly from first and second graders on equivalency by correspondence, $F(1,57) = 14.93$, $p < .003$; conservation of number, $F(1,57) = 7.11$, $p < .01$; and asymmetry by counting, $F(1,57) = 9.10$, $p < .004$. First graders performed significantly less well than second graders on equivalency by correspondence, $F(1,57) = 11.20$, $p < .002$; and asymmetry by correspondence, $F(1,57) = 6.26$, $p. < .02$" (pp. 731-732). An order analysis procedure was used to determine a hierarchical ordering of the tasks. The resulting diagram showed that "counting tasks were easier than correspondence tasks, while equivalency by correspondence proved to be the most difficult procedure for children in this study" (p. 732). Transitivity tasks were not included in the order analysis because the scores were spuriously high.

5. Interpretations

"The findings of this investigation support Brainerd's (1978) contention that conservation of number is an intermediate step in children's understanding of quantitative correspondence...(and) the finding that ability to determine asymmetric cardinal relations appear prior to appropriate judgments of equivalent relations when correspondence is the strategy used" (p. 732). The author discussed why the similar results from Brainerd's static correspondence and the collinear correspondence tasks differ substantially from the results obtained from conservation of number tasks. In the main, the children of the present study and Brainerd's failed to use a nonperceptual pairing strategy.

The author also discussed at length the reasons for the high transitive scores and for their exclusion from the order analysis. Forty percent of the children correctly answered the transitive question but failed to answer the asymmetric questions correctly. Since transitivity assumes an understanding of asymmetric relations, the transitive data were not analyzed. The author also discussed the limitations of the study: the conservative scoring and the wording of the prompts in the correspondence tasks.

Abstractor's Comments

Russac has attempted a difficult task, that of investigating hierarchies of learning or relative acquisition of mathematical concepts. The question of whether counting or correspondence is understood first is an important one for an understanding of how the concept of number is learned. This, of course, has implications for mathematics education and the teaching of early number concepts. There are, however, several questions this reader had about the study.

1. Was there a pilot study? If so, why wasn't the problem of the transitive question noticed then? Russac is to be commended for looking at the quality of the data before blindly analyzing it. Too many investigators fail to ask whether the raw data actually have meaning before applying the most sophisticated of analysis techniques. From the discussion of the problem, it appeared that the children were using the perceptual cue of length of the line of red dots to decide which of

the 7- or 9-dot sets had more dots. Using strips in which the relative number of dots conflicted with length-density cues may have changed the results.

In addition, a pilot study would have revealed the need for word cues in the correspondence tasks. As the author noted, children were told to "count" in the counting tasks, but there was no similar help such as "match" or "pair," in the correspondence tasks. These simple changes will improve the replicated study.

2. Were the reported F-ratios the results of post-hoc contrasts (Scheffe, Tukey)? In any case, the degrees of freedom for the F statistic should have been 2 and 57 rather than 1 and 57, since there were three groups of children. An incorrect reporting (or uncorrected typing) of simple statistics casts doubt upon the rest of the analysis.

3. What was the specific definition of "perceptual" and "nonperceptual"? From the context it appeared that a perceptual pairing meant one in which the dots or chips were on top of one another, in parallel rows next to one another, or in some regular spatial arrangement. Nonperceptual should mean that the child used pointing or some mental means of pairing the sets. Russac also included the particular "collinear correspondence" used in the experiment as a nonperceptual pairing. Actually the "collinear correspondence" seemed to be both perceptual and nonperceptual (assuming the abstractor gleaned the proper definition from the text). It was perceptual in that the chips were physically placed next to the dots; it was nonperceptual in that the child had to remember which chip was paired with which dot.

The results supporting Brainerd and differing from Piaget hinge entirely on the "collinear correspondence" task the children had to perform correctly to show understanding of equivalence by correspondence. Besides not knowing exactly what the adult wanted the child to do with the red dots or blue chips, the child may have been very frustrated with the admonition not to count nor to put the chips on top of the dots. As Russac mentioned when discussing this limitation, use of the words "match" or "pair" may have eliminated some of the problems. However, most of the problems seemed to occur because the children had never made arrays before, whereas the children appeared to have made correspondences in parallel rows since they had little difficulty in the pretransformation correspondence task.

Russac included the pretransformation correspondence task before the conservation of number task to attempt to determine the relation between that correspondence and the "collinear correspondence" task. The order analysis indicated that the pretransformation correspondence was acquired before conservation of number, which in turn is acquired before equivalence by correspondence. Since both correspondence tasks are comparing equivalent sets with only differing methods of correspondence, might not an alternate interpretation of the analysis be possible? Children in the early grades do many activities which involve pairing or matching equal sets. They also do many worksheets matching number symbols to pictures of sets. Almost all of these tasks usually involve a regular spatial arrangement of the sets. Thus, the order analysis may only reveal that equivalence by correspondence precedes conservation of number if the correspondence is a familiar one, while it is acquired after conservation of number if the particular correspondence required is an unfamiliar type. If the children of the study had experience with many types of correspondence tasks, the hierarchy of acquisition of the concepts of cardinal number may have been different.

Russac's results indicated that counting was acquired before correspondence and conservation of number, contrary to the results of Piaget. How much does "Sesame Street" influence the counting of American children? Is there any child who has not been indoctrinated with rote counting from the time he or she began watching television? Will mathematics education researchers ever be able to determine acquisitional relations between counting and correspondence?

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Abstract and comments prepared for I.M.E. by JOHN TARR, University of Northern Iowa.

1. Purpose

The purpose of this study was to determine the effect on students' basic skill development of using the innovative curriculum called Unified Science and Mathematics for Elementary Schools (MSMES). Six basic skill areas--reading comprehension, mathematics application, mathematics concepts, science, and social science--were included in the investigation. In the study USMES was used in elementary school grades which were grouped into three blocks: 2-4, 5-6, and 7-8. The effects were measured by pretesting in the fall and posttesting in the spring.

2. Rationale

USMES units are designed to engage elementary school students in long-range, complex investigations of real and practical problems taken from their school or community environment. Students are supposed to gain competence in problem solving in a manner that gives them an understanding of the problem-solving process as well as the acquisition of basic skills and concepts.

In the economic and political climate in which USMES was implemented, educators were forced to take cautious views of new programs, particularly if the programs might detract from efforts to teach basic skills. A significant portion of the evaluation of USMES involved assessing the effects of USMES on students' basic skill development.

The "back-to-basics" movement which swept the country during the 1970s, coupled with the relative ease of measuring basic skills objectives, likely account for the tendency to focus accountability programs narrowly on lower level cognitive goals, while excluding attention to the difficult-to-measure higher mental processes and elusive affective goals. The USMES curriculum did not escape the pressure to remain successfully accountable for students' basic skill development. USMES developers and evaluators had to seek evidence that the USMES students performed at least as well as non-USMES students in the basic skills.

3. Research Design and Procedures

To determine the effect of USMES on the development of basic skills, a study was made of USMES classes and non-USMES classes. USMES teachers from 15 geographic areas were selected to achieve a national sample of USMES classes representing a cross-section of grade levels, socioeconomic levels, and USMES units. Control classes came from non-USMES schools which were located in the same or neighboring communities as the USMES schools. The control classes were selected to match the USMES sample classes, one-for-one, on the bases of grade level, socioeconomic level, geographic area, and general features of the schools' programs. Although the investigation intended to use a sample of 40 USMES classes and 40 control classes, difficulties in data collection resulted in usable returns from at most 60 classes for any of the subtests.

Six subtests of the Stanford Achievement Test (SAT) series were used. Reading Comprehension and Mathematics Computation subtests were administered in all the classes. Mathematics Concepts and Science subtests were administered in half of the classes; Mathematics Application and Social Science subtests were given in the other classes. A scaled score was obtained for each student for each of four subtests the student took. The scaled score supposedly offered a system of inter-battery standard scores with comparability across levels for a test area. The unit of analysis was the classroom.

Pretests were given in late September and early October; posttests were given during the following May. Various levels of the test were employed, one per class. Grades were grouped into three blocks: 2-4, 5-6, and 7-8.

Thus the study involved three independent variables: treatment (USMES, non-USMES), grade (three blocks), and administration (Fall, Spring). It involved six independent variables--classroom means of scaled scores on each of six SAT subsets.

The data for each subtest were submitted to a repeated measure analysis of variance to investigate whether classes from either treatment group realized statistically significant gains in any of the six subtest areas from fall to spring. Also, across the six analyses, 24 interaction effects were tested.

4. Findings

Results from the six repeated measures analyses of variance are summarized in the following table. Complete ANOVA tables and tables of cell means and standard deviations from the analysis of each subtest are given in the final report to the National Science Foundation on the evaluation of USMES (Shann et al., 1975). These data were not included in this report.

Summary of Significant Results of Repeated Measures Analyses of Variance on Six SAT Subtests

Subtest	n^a	Treatment[b]	Grade[c]	Pre-Post[d]	Interactions
Reading Comprehension	60	N.S.	$p < 0.0001$	$p < 0.0001$	$G \times A^e$, $p < 0.05$
Mathematics Computation	58	N.S.	$p < 0.0001$	$p < 0.0001$	$G \times A^e$, $p < 0.0001$
Mathematics Application	22	N.S.	$p < 0.0001$	$p < 0.01$	none
Mathematics Concepts	31	N.S.	$p < 0.0001$	N.S.	none
Science	29	N.S.	$p < 0.0001$	$p < 0.01$	none
Social Science	25	N.S.	$p < 0.0001$	$p < 0.01$	$G \times A^e$, $p < 0.05$; $T \times A^f$, $p < 0.05$

[a] n = number of classes; class means were used as the unit of analysis

[b] USMES versus non-USMES

[c] Grades were grouped into three blocks: 2-4; 5-6; and 7-8.

[d] These were Fall and Spring administrations of the SAT subtests.

[e] Interaction of grade with test administration

[f] Interaction of treatment with test administration

There were highly significant increases ($p < 0.0001$) in all six subtest measures from the lower to higher elementary grades for both treatment groups. Also, significant increases from pretest to posttest administration for both treatment groups within each grade level were found for all but one subtest, Mathematics Concepts. There were no significant treatment differences. Although the observed treatment differences consistently favored USMES, these differences were not statistically significant.

Twenty of the 24 interaction effects tested were not significant. The analysis for Reading Comprehension and for Mathematics Computation subtest scores each produced a significant grade-by-administration interaction. In Reading Comprehension, the pre-post mean gains for the three grade blocks, 2-4, 5-6, and 7-8, were 12, 7, and 1 points, respectively. In Mathematics Computation, the pre-post mean gains by grade level were 12, 6, and 0.2.

The other two significant interaction effects were observed from the analysis of social science subtest scores. Although the USMES groups at all three blocks of grade levels showed gains from pretest to posttest, their rate of growth declined for the seventh and eighth grades. Changes in control group pre-to-post scores varied erratically by grade blocks--some no change, some higher, some declines.

5. Interpretations

With the exception of the two interaction effects from the analysis of the social science subtest scores, the results from the repeated measures analyses presented no surprises; they conformed solidly with the results one should expect.

The highly significant increases in all subtest measures from the lower to higher elementary grades for both treatment groups were consistent with expected growth patterns for these areas of achievement. One would expect seventh- and eighth-grade classes to score higher than second- and third-grade classes and, in this study, they did.

The significant growth patterns from pretest to posttest are what one would hope to find for both treatment groups. That is, one would expect scores to be higher in the spring than in the previous fall.

In this study, significant increases were found in all but one subtest, Mathematics Concepts.

Sophisticated analyses were not needed to ascertain that the national sample of USMES classes performed at least as well as their control matches on the six selected subtests. USMES students had not been deprived of instruction in the basic skills and, in some cases, they may have received more instruction than non-USMES students. Thus, in the mathematics and science skills area, USMES should not have been interfering with skill and concept development, but rather, adding to it. The results of this study are, therefore, not surprising.

The grade-by-test administration interaction effects noted for Reading Comprehension and for Mathematics Computation may be a function of a ceiling effect of these subtests and on lesser attention devoted to these subjects over the years of a school's program. Also, these two significant interaction effects mirror the growth curves for these basic skills. The two interaction effects from the analysis of the social science subtest scores were surprising. No explanation was offered.

Abstractor's Comments

Mary Shann has clearly presented the problem, methodology, findings, and implications of the study of the effect of USMES on the basic skills.

One might ask if this study was necessary. In a time when "basics" are stressed, everything else is questioned. Perhaps in another era we might question whether instruction in "the basics" interferes with the students' competence in complex problem solving. Do students in a particular mathematics or science program perform at least as well as other students on tests measuring problem-solving competence? For the 1970s, however, the study was necessary and its results should allay the fears of those protecting the basic skills.

The study appropriately used the classroom as the unit of analysis and care was taken to select a large enough sample so that the number in each cell was sufficient. The control classrooms appear to have been matched carefully to the USMES classrooms. Is it possible, however, that teachers electing to use USMES materials differ in some way from those who do not elect to use USMES materials? Could teachers who had volunteered to use USMES materials have been used as controls?

The results of the study suggest that USMES is "safe"; performance in the basic skills was not lowered. One might question what the implications would have been if the basic skills scores were significantly lower for USMES classes. What price would we be willing to pay for improved competence in complex problem solving?

Time given to instruction in the basic skills apparently was affected very little by the introduction of USMES units. Where then did the time come from? Since the length of the school day likely remained the same, was USMES instructional time taken from music, art, or physical education?

The investigator raised a question about the scaling procedures used. The USMES evaluators were concerned that the formulas given by the SAT test developers had not achieved comparability of translated, scaled scores across test levels for a given test area. This may have been responsible for the lack of homogeneity of variance experienced in this study. Although it was beyond the scope of the USMES evaluation to test this hypothesis, it deserves further study. The investigator suggests how such a study might be structured.

The need for more objective instruments to measure problem-solving skills was noted by the investigator. New instrument development for complex problem solving has become a thrust of the USMES evaluation team's efforts since a valid accounting of the program's worth is dependent on a valid measure of complex problem solving. The development of new tests to measure students' achievement in various aspects of problem solving deserves the concerted attention of the evaluation community.

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